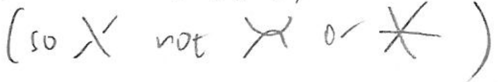
Knots

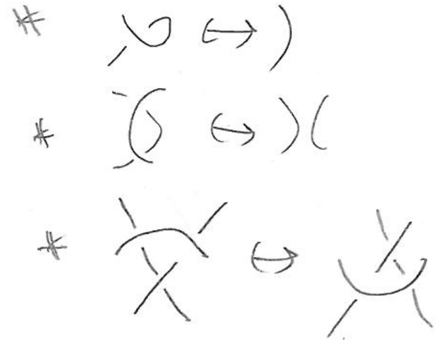
- An <u>n component link</u> L is a subset of $\mathbb{R}^3$ consisting of n disjoint piecewise-linear closed curves.
  If $n=1$, L is a <u>knot</u>

- $L = L'$ equiv if $\exists$ orient. preserving p.l. homeo $h: \mathbb{R}^3 \to \mathbb{R}^3$ st $h(L) = L'$

- A <u>knot invariant</u> is a function on links s.t.
  $L = L' \implies f(L) = f(L')$

- <u>Example</u> # cpts.
  $O \neq OO \overset{?}{=}$ 

A link diagram is a "drawing of the link on the plane"
- more formally
  project $\mathbb{R}^3 \to \mathbb{R}^2$
  mark on crossings and make transversal, 2-strands.
  (so )

- <u>Reidemeister's Theorem</u>
  Two links are equivalent $\iff$ their diagrams are related by the Reidemeister moves
  * planar isotopy
  * 

Jones polynomial

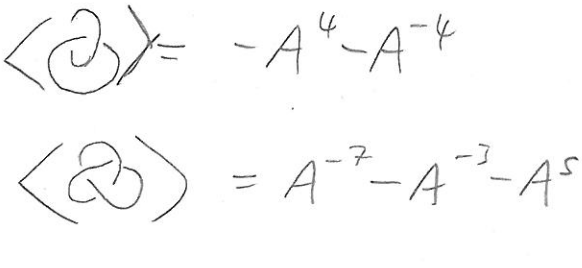
<u>Kauffman bracket</u>

$\langle D \sqcup O \rangle = d \langle D \rangle$

$\langle O \rangle = 1$

where $d = -A^{-2} - A^{2}$

<u>Example</u>

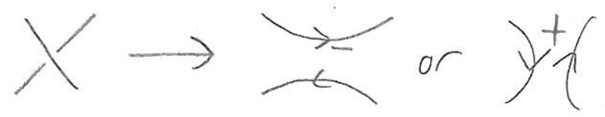


<u>Jones polynomial</u> For L oriented

$$V(L) = \left((-A)^{-3w(D)} \langle D \rangle \right) \Big|_{t^{\frac{1}{2}} = A^{-2}}$$

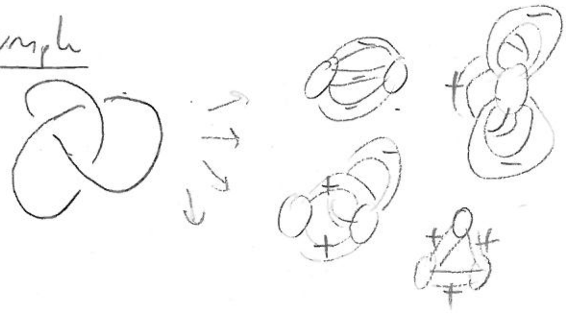

<u>writhe</u>

Ribbongraphs of a link diagram

- 

Link diag   signed arrow pres.

example



- All-A is the ribbongraph with all −ve signs

- Tait graphs those that follow a c.b. colouring

(plan

Thm

$$\langle D\rangle = d^{k(A)-1} A^{n(A)-r(A)} R(A; -A^4, A^{-2}d, d^{-1})$$

pf:

$$su(D) = \sum_{A\subseteq E} A^{(\text{edges out})} A^{-1(\text{edges in})} d^{(\text{cycles}-1)}$$
$$= A^{\#\text{edges}} d^{-1} \sum_{A\subseteq E} A^{-2|A|} d^{f(A)} = \dots$$

Corollary: If D is alternating

$$V(L;t) = (-1)^{w(D)} t^{(3w(D) - n(A) + r(A))/4} (-t^{\frac{1}{2}} - t^{-\frac{1}{2}})^{k(A)-1} T(A; -t, -\tfrac{1}{t})$$

Observations

- Not all ribbon graphs describe link diagrams.
- Different link diagrams can give rise to the same set of ribbon graphs.

Q:

- How are RGs of link diags related?
- Which RGs describe links?
- How are diagrams w/ same set of RGs related?

Thm

G and G' represent same diagram

$\iff$ partial duals

where duality reverses sign

$$q \to \begin{cases} q & \text{if } e \notin A \\ -q & \text{if } e \in A \end{cases}$$

pf:

partial dual
arrow presentations □

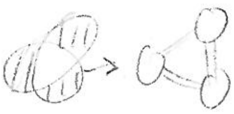
Thm:

G reps a link diag

$\iff$ PD of plane graph.

pf:

Every link diag is represented by a plane ribbon graph.



Corollary

G reps link diag

$\iff$ admits a plane-biseparation

$\iff$ no  minors

□

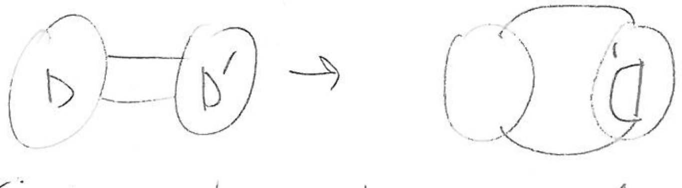
- Suppose diagrams D and D' give rise to G.
  How are D and D' related?

- recover link diagrams from G
  - Take a plane partial dual G^A
  - 
  - gives L_{G^A}

- D, D'
  - Take 2 partial duals G^A, G^B
  - Draw links

- But G^A, G^B both plane
  $\Rightarrow$ related by dual summand moves

$\Rightarrow$ D D' related by sequence of



Cor: Isotopy class rep by unique set of RGs.