MINI-COURSE: NON C^*-EXACT GROUPS

GOULNARA ARZHANTSEVA

ABSTRACT. A countable discrete group G is C^*-*exact* or simply, *exact*, if its reduced C^*-algebra $C^*_r(G)$ is an exact C^*-algebra (i.e. if taking the minimal tensor product with $C^*_r(G)$ preserves short exact sequences of C^*-algebras). Equivalently, G is exact if it admits an amenable action on some compact Hausdorff space. Exact groups are also said to be boundary amenable, amenable at infinity, to have Guoliang Yu's property A or to be coarsely amenable. The exactness is viewed as a weak amenability type condition. All amenable groups, linear groups, Gromov's hyperbolic groups, groups with finite asymptotic dimension, and many other familiar groups are known to be exact. In contrast, constructions of non-exact groups are rare and technically quite involved. We will discuss such constructions, indicate applications, and suggest some open problems.

Pre-requisites: basic undergraduate knowledge in combinatorial group theory (e.g. free groups, presentations of groups by generators and relators, Cayley graphs), graphs, and C^*-algebras.

Reference books.

For more details on basic concepts of combinatorial group theory one can consult:

Lyndon, Roger C.; Schupp, Paul E. Combinatorial group theory. Reprint of the 1977 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001. xiv+339 pp. ISBN: 3-540-41158-5

For a recent account on C^*-algebras one can consult:

Brown, Nathanial P.; Ozawa, Narutaka C^*-algebras and finite-dimensional approximations. Graduate Studies in Mathematics, 88. American Mathematical Society, Providence, RI, 2008. xvi+509 pp. ISBN: 978-0-8218-4381-9

For the discussion on exact and non-exact groups in the context of large scale geometry on can consult:

Nowak, Piotr W.; Yu, Guoliang Large scale geometry. EMS Textbooks in Mathematics. European Mathematical Society (EMS), Zrich, 2012. xiv+189 pp. ISBN: 978-3-03719-112-5

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1. Group C^*-algebras

2. Small cancellation theory

3. Constructions of infinite monster groups

4. Applications

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Universität Wien, Fakultät für Mathematik, Oskar-Morgenstern-Platz 1, 1090 Wien, Austria.

E-mail address: goulnara.arzhantseva@univie.ac.at